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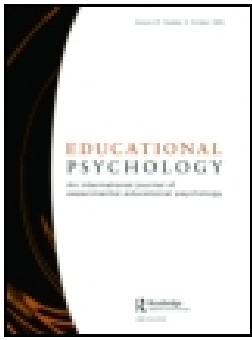
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A distinction between working memory components as unique predictors of mathematical components in 7–8 year old children

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ABSTRACT

Despite evidence for the involvement of working memory in mathematics attainment, the understanding of its components relationship to individual areas of mathematics is somewhat restricted. This study aims to better understand this relationship. Two-hundred and fourteen year 3 children in the UK were administered tests of verbal and visuospatial working memory, followed by a standardised mathematics test. Confirmatory factor analyses and variance partitioning were then performed on the data to identify the unique variance accounted for by verbal and visuospatial working memory measures for each component of mathematics assessed. Results revealed contrasting patterns between components, with those typically visual components demonstrating a larger proportion of unique variance explained by visuospatial measures. This pattern reveals a level of specificity with regard to the component of working memory engaged depending on the component of mathematics being assessed. Implications for educators and further research are discussed.

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Introduction

Mathematics is a very heterogeneous concept, including several different sub-domains. Dating back to the prehistoric times of the hunter-gatherer, the use of mathematics in the forms of number, magnitude, and form can be seen (Boyer & Merzbach, 2011; De Cruz, 2006); however, the term itself is not used until the time of the Greeks. Since its first use by the Ancient Greeks (Boyer & Merzbach, 2011), mathematics has been used as an umbrella term, seemingly accounting for pure arithmetical concepts, as well as other more specific concepts, such as geometry. Boyer and Merzbach (2011) describe how the term was coined by the Pythagoreans, and used by those who first began to study mathematics for its own sake. In literature surrounding these times, one can clearly see a distinction between arithmetical and geometrical mathematics.

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This sharp distinction between arithmetic and geometry was maintained for several centuries. For example, in the medieval age, arithmetic and geometry were distinguished and, alongside music and astronomy, were included in the so-called *quadrivium*, encompassing these four ‘mathematical’ subjects (Grant, 1999). Nowadays, curricula around the world have somewhat abandoned this distinction and we usually refer to mathematics, although a variety of different forms of mathematics exist and seem to be very different from one another.

Different predictors of mathematics performance have been identified but – among several others – working memory, a system for the short-term storage and manipulation of information, has been repeatedly associated with several different mathematic skills. It has been shown that working memory predicts performance on tests of approximate mental addition (Caviola et al., 2012, 2016; Kalamian & Le Fevre, 2007; Mammarella et al., 2013), written subtractions (Caviola et al., 2016, 2018), number facts (Steel & Funnell, 2001), multi-digit operations (Heathcote, 1994), magnitude representation (Pelegrina et al., 2015), arithmetical problems (Passolunghi & Siegel, 2001; Passolunghi & Mammarella, 2010; Rasmussen & Bisanz, 2005), quantitative central conceptual structures (Morra et al., 2019), and geometrical achievement (Giofrè et al., 2013, 2014). Importantly, working memory is a generic term, for which we also see alternative models.

Several alternative working memory models have been proposed, but the classical tripartite working memory model (Baddeley & Hitch, 1974), which includes a central executive, responsible for controlling resources and monitoring information, and two domain-specific modules for either verbal or visuospatial information, tends to be one of the most well-known (see Baddeley, 2000 for a review). Other accounts postulate the existence of a sharp difference between a working memory factor, which requires cognitive control to a large extent, and a short-term memory factor, which requires less cognitive control (i.e. fewer attentional resources; Kane et al., 2004). Finally, there is a domain-specific factors model, only distinguishing between verbal and visuospatial modalities (Shah & Miyake, 1996). The distinction between verbal and visuospatial working memory has recently received broader attention and might be of particular importance when considering mathematics as it aligns well with the historical argument that geometry is distinct from arithmetic, dealt with by visuospatial and verbal working memory, respectively, given the nature of the requirements of each.

Only a few studies consider the relationship between verbal and visuospatial working memory in mathematics or in typically developing children. The literature is rife with debate regarding the specific contributions to academic performance in both typically and atypically developing children (e.g. Alloway & Alloway, 2010; Geary et al., 2004; Szűcs et al., 2013). Studies have found evidence in support of the stronger influence of visuospatial working memory (e.g. Caviola et al., 2014; Clearman et al., 2017; Holmes et al., 2008; Li & Geary, 2017), however, evidence for the influence of verbal working memory can also be found (e.g. Hitch & McAuley, 1991; Wilson & Swanson, 2001), particularly verbal-numeric working memory (see Raghubar et al., 2010 for a review). Such diverse findings, however, might be attributable to the particular mathematical tasks used in different studies, and it appears plausible to hypothesise that different mathematical subdomains might require verbal and visuospatial working memory resources to a different extent.

The particular relation of the components of working memory to the components of mathematics is as yet a relatively under-researched topic, with much of the literature concerning the relationship between working memory components and mathematics performance as a whole. These particular relationships are not considered in recent meta-analyses, for example, by Friso-van den Bos et al. (2013) and Peng et al. (2016). Whilst there have been studies investigating the relationship between working memory and particular elements of mathematics (e.g. arithmetic: Ashkenazi et al., 2013; Caviola et al., 2012; Passolunghi & Cornoldi, 2008, or word problem solving: Swanson & Beebe-Frankenberger, 2004; Swanson & Sachse-Lee, 2001; Zheng et al., 2011), this remains an area that requires development. Research into the relationship between working memory and geometry has also received attention (e.g. Giofrè et al., 2013, 2014), as has its relationship with mathematical difficulties (Andersson & Lyxell, 2007; D'Amico & Guarnera, 2005; McLean & Hitch, 1999; Passolunghi & Cornoldi, 2008; Szűcs et al., 2013).

A more intricate understanding of the relationships between working memory and the components of mathematics is fundamental before future work can begin on developing interventions targeting children vulnerable to mathematics difficulties. This paper aims to further the debate discussed above by highlighting the differential contributions of components of working memory to different forms of mathematics. In this study, working memory will be divided into verbal and visuospatial components, whilst arithmetic will comprise using and applying mathematics, counting and understanding numbers, knowing and using number facts, and calculating. Geometry will consist of understanding shape, and handling data in order to encompass tasks that are inherently more visual in nature. These tasks rely heavily on diagrams and mental images of space, hence are intuitively more likely to draw on the visuospatial component of working memory. By assessing each of these areas with regard to the relative contributions of verbal and visuospatial working memory, it will be possible to understand more specifically how mathematics and working memory are related, as well as where to target mathematics interventions for the greatest effect. This analysis is performed on a data set previously analysed in Allen et al. (2020), which demonstrates the strongest unique influence of verbal-numeric working memory on mathematics, followed by spatial-simultaneous working memory (spatial working memory tasks during which all to-be-remembered information is presented simultaneously). This paper seeks to further this understanding to address how the balance of influence identified may be affected by the area of mathematics in question. It is important to note that no overlapping analyses are reported in either paper. We hypothesise that visuospatial working memory will be more influential in geometry due to the inherently visual nature of the tasks, whilst verbal working memory will remain more influential in arithmetic tasks since verbal working memory seems to be involved in tasks requiring fact recall and basic mathematical skills.

Method

Participants

The sample initially included 214 7–8-year old children. Some children were absent during the second administration and so were excluded from the final sample.

The final sample included a total of 197 children (95 males and 102 females, $M_{\text{age}} = 95.99$ months, $SD = 3.63$). An opportunity sample of Year 3 pupils in each of the five schools was gathered, using opt-out parental consent to reduce bias in the sample (Krousel-Wood et al., 2006). The study was approved by the School of Education Ethics Committee at the University of Durham. Parental consent was assumed if opt-out forms were not returned. Children with a diagnosis of a special educational need, including intellectual disability, or neurological or genetic disorder, were not included in the study. Children classed as low functioning or 'gifted' are routinely included in typical classes in the UK and were not therefore excluded from our sample.

Procedure

All children were tested individually, in a quiet area of their school. Measures were administered in a randomised order, so as to account for any order effects, however, the size of the grids used in the derived measures of visuospatial working memory were administered in a fixed order (3×3 then 4×3 , and 4×3 then 4×4 , for sequential and simultaneous, respectively). A correlational design was used to explore the relationships between visuospatial working memory and maths performance. Working memory measures were administered as per the administration instructions provided with the Working Memory Test Battery for Children (WMTB-C), in their original format. Additional visuospatial measures were derived for the purpose of the study, for which administration procedures paralleled those set out for standardised measures, however, were presented using a Windows laptop computer, as opposed to in physical form. The battery of measures used was chosen in order to ensure a fully crossed model for each type of verbal and visuospatial working memory. The mathematics test was presented in paper format, however, children could ask for questions to be read aloud in order to not place children of lower reading ability at a disadvantage.

Measures

Verbal Working Memory

Working memory test battery for children (WMTB-C). Three subtests of the WMTB-C were administered: digit recall (children recall a list of digits presented to them verbally), backwards digit recall (children recall a list of digits presented to them verbally in reverse order), and counting recall (children count aloud the number of dots on a page then recall the list of totals, in the correct order, once all pages in the sequence have been counted). All subtests were administered in accordance with the instructions set out for the WMTB-C, with items presented at a rate of one item per second. Trials were administered in blocks of six trials of the same length. Following four correct trials, testing moved on to the next block. Testing was discontinued following three mistakes within one block, unless this was the first block of trials, in which case the previous block was administered to ascertain the child's span score. A raw score, standard score, and span score was recorded for each child on each subtest.

Visuospatial Working Memory

Children were presented with three visuospatial working memory tasks (simultaneous, sequential without order during recall, and sequential with order during recall). For simultaneous and sequential without order tasks, a grid was presented containing dots. The dots were either presented all at the same time (simultaneous) or one at a time (sequential) for 3 s and 1 s each, respectively. Children were required to observe the positions of the dots and recall these positions following the removal of the stimulus. For sequential visuospatial working memory with order, the block recall subtest from the WMTB-C was employed.

Mathematics

Access mathematics test (AMT). The AMT was employed as a standardised measure of mathematics, available for use with children between the ages of 6 and 12 years. As such it provides a comprehensive profile of how children perform when faced with different aspects of maths. Further, the same measure can be given to older children in order to understand how this relationship with working memory may develop over time. The AMT is aligned to the areas of maths taught on the curriculum, hence providing a valid measure whereby performance on the test demonstrates likely performance on Government-prescribed mathematics tests. 'Children were read the instructions set out for the AMT, which included a time limit of 45 minutes, clarification of where to write their answer on the paper, and explanation that workings are allowed on the paper, providing their answer is clearly written in the correct space. Typical test conditions were adopted throughout. Children were permitted to request questions be read aloud to them should they have difficulties so as not to disadvantage those with weaker reading abilities, however, no further explanation of the question, or what was required, was given. No discontinuation rule was employed as children were instructed to complete as many questions as they could, but that questions were also included for children much older than they were so not to worry if they could not complete them all' (Allen et al., 2020, p. 241). All mathematics testing was carried out after the completion of all working memory testing. The two testing phases were on different days for all children. The components of mathematics included were as follows: using and applying mathematics (8 questions), counting and understanding number (12 questions), knowing and using number facts (8 questions), calculating (8 questions), understanding shape (8 questions), and handling data (8 questions) ($\alpha = .96$ and $\alpha = .97$ for test forms A and B, respectively).

Questions included those concerning using and applying mathematics (e.g. 'circle the two addition facts that give the same answer'), counting and understanding number (e.g. 'circle the number that is nearest in value to 75'), knowing and using number facts (e.g. 'what is double 32?'), calculating (e.g. 'complete this calculation and show the remainder: $659 \div 5 = \underline{\hspace{1cm}} \text{ remainder } \underline{\hspace{1cm}}$ '), understanding shape (e.g. 'a tetrahedron has four corners and four faces. How many edges does a tetrahedron have?' [a picture of a tetrahedron is included for reference] and see [Figure 1\(a\)](#) for a further example), and handling data (see, e.g. [Figure 1\(b\)](#)). Arithmetic tasks are presented in a variety of ways and ask children to do a number of things from completing calculations to selecting from multiple-choice options.

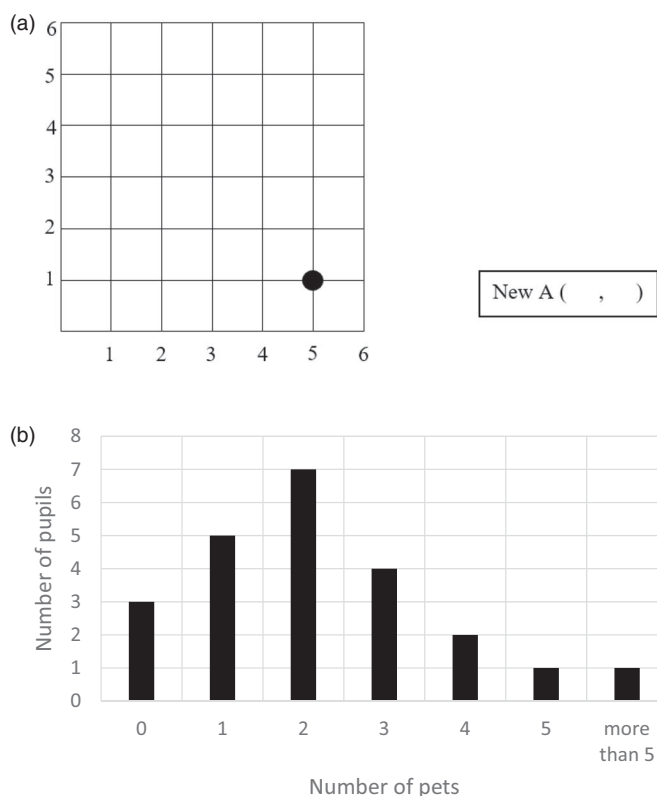


Figure 1. Diagrams showing example questions for understanding shape and handling data sub-scales of AMT (a and b, respectively). (a) The point A is moved two squares to the left and four squares up. Write the coordinates of the new point A. (b) The bar chart, from a spreadsheet, shows the number of pets each pupil owns. How many pupils own 3 or more pets?

Geometric tasks also concern a number of skills, with handling data questions mainly concerning the construction and interpretation of graphs and charts, and understanding shape tasks including tasks encompassing a range of skills such as transformations and properties of shapes.

Data analysis

The R program (R Core Team, 2018) with the 'lavaan' library (Rosseel, 2012) was used. Model fit was assessed using various indices according to the criteria suggested by Hu and Bentler (1999). We considered the chi-square (χ^2), the standardised root mean square residual (SRMR), the root mean square error of approximation (RMSEA), and the comparative fit index (CFI). This data set has been previously analysed in Allen et al. (2020), however, previous analysis was concerned only with the relationship between verbal and visuospatial working memory and mathematics, but without distinguishing between different mathematic subcomponents. Analyses in the variance partitioning section were performed using the latent correlation matrix for each model. This matrix was used for calculating the R^2 for multiple regressions using the 'mat.regress' function available for the 'psych' package (Revelle, 1970; see Cohen et al., 2003 for the statistical rationale).

Results

Descriptive statistics

Descriptive statistics and age-covary correlations are provided in Table 1. Age-covary values were obtained using regressions in which age was entered as a predictor and residuals, controlling for age, were obtained. Age-controlled values were then used for all subsequent analyses (see Allen et al., 2020; Giofrè & Mammarella, 2014; for a similar procedure).

Confirmatory factor analysis (CFA)

In model (CFA00) the factorial structure of working memory, including two components (verbal and visuospatial) was evaluated, results indicated that the fit for this model was adequate (see also Giofrè et al., 2018 for a similar result) and this factor structure has therefore been maintained for subsequent analyses. Successively, we performed a series of CFA analyses, one for each component of mathematics, using the overall scores, and following the general guidelines for SEM with observed indicators (Kline, 2011). Importantly, the fit index of each individual model was good, indicating that a distinction between verbal and visuospatial working memory was adequate. We decided to use CFA because we were mainly interested in the relationship between constructs at the latent level (i.e. verbal vs. visuospatial working memory). Moreover, CFA allows a more precise estimate of the relationship between the construct of interest, reducing problems related to the unreliability of individual predictors (Kline, 2011). Several different models for each individual task were tested in order to obtain baseline estimates (i.e. correlation matrices) to be used in subsequent analyses (Table 2).

Variance partitioning

In the final set of analyses, starting from the correlation matrices obtained in the CFA, we used variance partitioning to examine the unique and shared portion of the variance of mathematics explained by the verbal and visuospatial factors. A series of regression analyses were conducted (see Chuah & Maybery, 1999; Giofrè et al., 2018 for a similar procedure). To derive the R^2 components for the various tasks, a number of regression analyses must be conducted (Chuah & Maybery, 1999). In this specific case, if verbal working memory is included in the first step (Model 1), while spatial working memory is included in the second step (Model 2), the resulting ΔR^2 corresponds to the unique contribution of spatial working memory over and above the effect of verbal working memory (i.e. R^2 of Model 2 – R^2 of Model 1). Vice versa, if spatial working memory is included in the first step, while verbal working memory is included in the second step, the resulting ΔR^2 corresponds to the specific contribution of spatial working memory over and above the effects of verbal working memory. Finally, the shared variance between verbal and visuospatial working memory can be obtained by subtracting the unique portions of variance uniquely explained by verbal and visuospatial working memory from the overall portion of the variance explained when these indicators are included simultaneously in the equation (i.e. the overall R^2 – unique variance of both verbal and visuospatial working memory).

Table 1. Means and standard deviations, with age-controlled correlations for each measure.

	1	2	3	4	5	6	7	8	9	10	11	12
1. Simultaneous	1											
2. Sequential	0.53*	1										
3. Block recall	0.43*	0.35*	1									
4. Counting recall	0.34*	0.26*	0.24*	1								
5. Backward digit	0.36*	0.29*	0.26*	0.42*	1							
6. Digit recall	0.13	0.10	-0.03	0.29*	0.25*	1						
7. Understanding and applying mathematics	0.32*	0.24*	0.22*	0.35*	0.29*	0.10*	1					
8. Counting and understanding number	0.30*	0.24*	0.12	0.33*	0.31*	0.18*	0.48*	1				
9. Knowing and using number facts	0.42*	0.29*	0.18*	0.30*	0.34*	0.16*	0.51*	0.62*	1			
10. Calculating	0.29*	0.23*	0.16*	0.22*	0.24*	0.09	0.46*	0.59*	0.65*	1		
11. Understanding shape	0.34*	0.18*	0.03	0.25*	0.12	-0.05	0.38*	0.41*	0.38*	0.36*	1	
12. Handling data	0.33*	0.24*	0.14*	0.21*	0.13	0.11	0.43*	0.51*	0.47*	0.40*	0.20*	1
M	48.75	34.26	21.86	16.47	10.56	26.69	1.77	3.18	2.17	1.66	0.9	1.36
SD	11.84	7.84	3.55	3.92	2.93	3.14	1.24	1.89	1.43	1.24	0.98	1.18

* $p < .05$.

Table 2. Fit indices for different CFA models.

Model	χ^2 (df)	<i>p</i>	RMSEA	SRMR	CFI	AIC
CFA00 WM	8.36 (8)	.371	0.021	0.041	0.997	6858
CFA01 UA	10.657 (12)	.559	0.000	0.038	1.000	7458
CFA02 CN	10.608 (12)	.563	0.000	0.038	1.000	7633
CFA03 NF	11.377 (12)	.497	0.000	0.038	1.000	7511
CFA04 Ca	9.103 (12)	.694	0.000	0.036	1.000	7480
CFA05 Sh	27.756 (12)	.006	0.082	0.056	0.934	7394
CFA06 HD	11.518 (12)	.485	0.000	0.039	1.000	7464

χ^2 : chi-square; RMSEA: root mean square error of approximation; SMSR: standardised root mean square residuals; CFI: comparative fit index; AIC: Akaike information criterion. UA: using and applying mathematics; CN: counting and understanding number; NF: knowing and using number facts; Ca: calculating; Sh: understanding shape; HD: handling data.

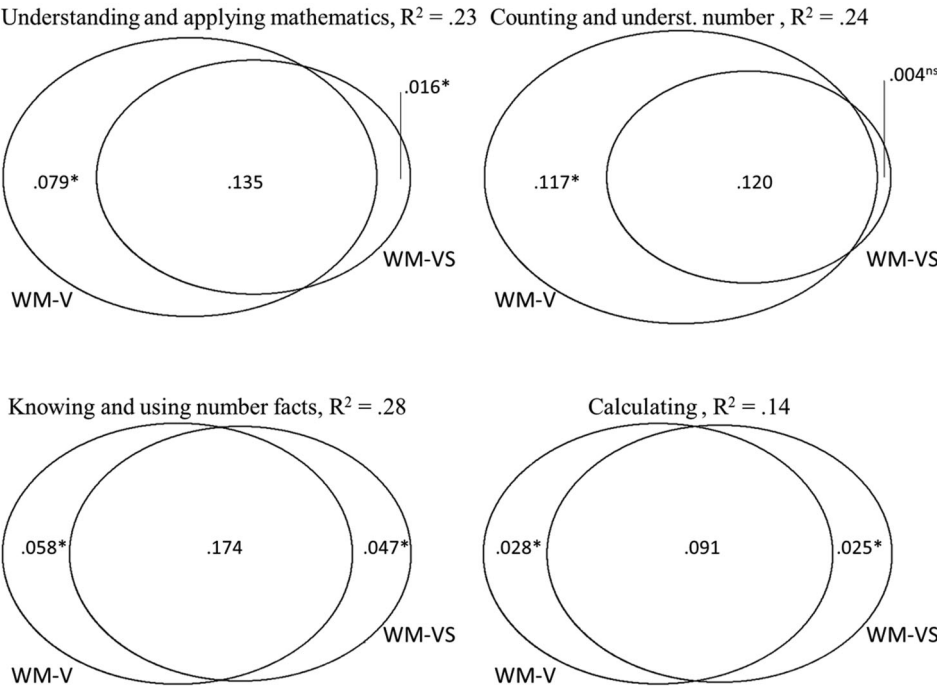


Figure 2. Venn diagrams indicating the shared and unique variance explained in using and applying mathematics, counting and understanding number, knowing and using number facts, and calculating by visuospatial and verbal factors. The overall area is proportional within each task, but not across tasks. * = $p < .05$, calculated using semi-partial correlations. ^{ns} = not statistically significant, calculated using semi-partial correlations.

The variance partitioning analysis is particularly useful for distinguishing shared variance, i.e. the portion of the variance that is common to two or more predictors, and unique variance, i.e. the portion of the variance which is uniquely predicted by one indicator (verbal or visuospatial working memory in this case).

Some mathematics components, i.e. using and applying mathematics, counting and understanding numbers, are more heavily influenced by verbal working memory (Figure 2), whereas understanding shape and handling data demonstrate a larger visuospatial component (Figure 3).

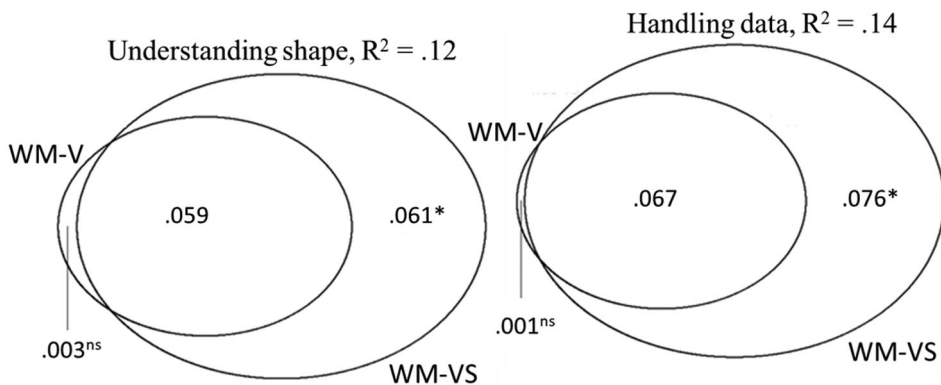


Figure 3. Venn diagrams indicating the shared and unique variance explained in understanding shape, and handling data by visuospatial and verbal factors. The overall area is proportional within each task, but not across tasks. * = $p < .05$, calculated using semi-partial correlations. ^{ns} = not statistically significant, calculated using semi-partial correlations.

Discussion

The principal aim of this paper was to further understand the individual contributions of verbal and visuospatial working memory on several distinct aspects of mathematic achievement. Previous evidence tends to be limited to the analysis of the overall performance in mathematics while an intricate understanding of the relationships between working memory and the components of mathematics might have important implications on developing interventions targeting children with mathematics difficulties.

From the venn diagrams, it is evident that the percentage variance accounted for by working memory components varies depending on the element of mathematics in question. Consistently, the largest percentage is accounted for by the shared variance between verbal and visuospatial measures. With regard to using and applying mathematics and counting and understanding number, the next largest percentage is accounted for by verbal measures (7.9% and 11.9%, respectively). This can be interpreted as the amount of variance in these components of mathematics accounted for by verbal-numeric measures over and above the influence of all other variables measured. Such a relationship is in line with previous literature relating verbal-numeric measures to mathematics performance (see Raghubar et al., 2010).

One potential explanation for this relationship emerging for these components is the mental maturation of the children, here accounted for by age. Age appears critical when considering the relationship between visuospatial working memory and mathematics (Holmes and Adams, 2006; Holmes et al., 2008; Li & Geary, 2013) with a stronger relationship demonstrated with younger children. Hence, by the age of the children involved in this study, there may have been a shift to verbal strategies, as suggested by Soltanlou et al. (2015). Further, the suggestion of a cyclical relationship between visuospatial working memory and verbal working memory conforms to the assumption that visuospatial working memory relates more strongly to the acquisition of new skills (Andersson, 2008). Consequently, once children reach 7–8 years of age, they may have sufficient experience with the material required for answering questions of this nature that they do not need to rely on visuospatial supports.

When considering knowing and using number facts and calculating, a different relationship is evident. Whilst verbal-numeric measures technically continue to explain the second greatest portion of unique variance, this difference with visuospatial measures is negligible. One potential explanation for the influence of visuospatial measures on these tasks is the format of the questions. All mathematical questions were presented to children in written format; a format which may inherently engage the visual component (e.g. Wong & Szűcs, 2013). This may be particularly potent for measures of calculating as question format has been shown to influence strategy choice (Katz et al., 2000). Strategy choice may be more or less 'fixed' in different areas of mathematics, depending on children's familiarity and experience with the component. For areas such as calculating that are taught from an early age and where children have more experience, they may have a greater variety of strategies at their disposal which may be a better or worse fit for a question depending on the style of presentation. However, this is speculation in this case as it is beyond the realms of this paper to answer this question. Whilst this may affect written over verbal question presentations, the influence of strategy choice dependent on the layout of written questions (as shown by O'Neil Jr & Brown, 1998) should be minimal in this study as questions were presented in a variety of ways, e.g. multiple-choice, open questions. Future research should be mindful of this influence and could seek to investigate how the layout of the questions themselves may influence method choice, and thus the extent of the involvement of visuospatial working memory (Cragg & Gilmore, 2014).

Perhaps the starkest difference is present between these previous four components of mathematics and the understanding shape and handling data components. In these cases, a shift towards a much larger influence of visuospatial working memory is clear. This shift is as expected, given the visual nature of the tasks, and confirms the heavy involvement of visuospatial working memory in those tasks wherein visual information is paramount to success. Previous work has identified a similar relationship between visuospatial working memory and geometry (e.g. Kyttaelae & Lehto, 2008), with complex visuospatial working memory tasks demonstrating predictive power for academic achievement in geometry (Giofrè et al., 2013), as well as accounting for group differences in performance in geometry between typically developing children and those with a non-verbal learning difficulty (Mammarella et al., 2013). Our findings mirror these results, suggesting that further research should be conducted in this area to determine the specific nature of the relationship between visuospatial working memory and shape in order that preventative and/or restorative measures can be devised.

Importantly, evidence of the distinct contributions of elements of working memory to geometry performance has been shown for both typically (Bizzaro et al., 2018; Giofrè et al., 2013, 2014) and atypically developing (Mammarella et al., 2013) children, distinct from measures of pure arithmetic. The aforementioned work revealed that academic achievement in geometry was influenced by working memory, with exaggerated differences between typically and atypically developing children in terms of Euclidian geometry as a result of visuospatial working memory performance.

In a meta-analysis focussed on working memory updating and its relation with mathematics, it was found that the comparison between verbal and visuospatial working memory subdomains was in fact statistically significant (see Table 2 of the original report), albeit modest in terms of magnitude (Friso-van den Bos et al., 2013). Intriguingly, arithmetic, counting and conceptual skills showed lower correlations with visuospatial updating. It is worth noting, however, that Peng et al. (2016) in their recent meta-analysis did not find significant differences between verbal and visuospatial working memory regarding their relationship with mathematics. It is also worth mentioning that concerning geometry, these results were based on a very limited number of observations, i.e. seventeen effect sizes for visuospatial working memory and sixteen for verbal working memory, with a very small number of studies overall, thus making it hard to test other moderating effects (e.g. the school year). Taking these results overall, we can confirm that more research is needed, confirming the importance of evaluating the unique contribution of verbal and visuospatial working memory on each mathematical subdomain.

It is important to note that the relationship identified here, specific to geometry, shows some variation from relationships identified between pure arithmetic components and working memory (e.g. arithmetic: Ashkenazi et al., 2013; Caviola et al., 2012; Passolunghi & Cornoldi, 2008, word problem solving: Swanson & Beebe-Frankenberger, 2004; Swanson & Sachse-Lee, 2001; Zheng et al., 2011, mathematical difficulties: Andersson & Lyxell, 2007; D'Amico & Guarnera, 2005; McLean & Hitch, 1999; Passolunghi & Cornoldi, 2008; Szűcs et al., 2013). Here, we see a greater contribution made by verbal working memory (e.g. Wilson & Swanson, 2001) over that contributed by visuospatial working memory (e.g. Caviola et al., 2014; Clearman et al., 2017; Holmes et al., 2008; Li & Geary, 2017), which is not entirely unexpected, given the types of questions associated with assessments of each type of mathematics.

With regard to the alternative models described in the introduction, the findings refute the model by Kane et al. (2004) as we see domain-specific contributions despite the inclusion of working memory measures. This model postulates that only short-term memory is domain-specific, whilst working memory tasks represent a domain general executive component, though this does not seem to be the case with the results here. In contrast, the results do seem to align with the domain-specific findings of the model by Shah and Miyake (1996), however, their measure of verbal working memory involved reading span. Hence, we cannot be sure our findings have not been influenced in some way by the numeric component of the verbal tasks used, which may have increased the strength of the relationships with verbal working memory (see Raghubar et al., 2010 for a review of the influence of verbal-numeric tasks).

In conclusion, this paper highlights a differential relationship between working memory tasks and mathematics attainment, dependent on the component of mathematics in question. Verbal-numeric tasks appear to be more predictive of performance on tasks more closely linked to factual recall and basic mathematical skills. In contrast, we see a stronger influence of visuospatial working memory in components of mathematics with a clear visual element: understanding shape and handling data. This is also in line with evidence indicating that different brain areas are activated in tasks requiring the manipulation of number or space (Arsalidou & Taylor, 2011; Kanayet et al., 2018).

To sum up, mathematics is a very broad term which encompasses several different domains, which are probably distinguishable. This should be taken into account in future research, in fact talking about ‘mathematics’ might not make sense, and research should focus on a more in-depth understanding of different mathematics subdomains. Finally, practitioners working with children with mathematical difficulties should try to understand the causes of these difficulties, trying, for example, to understand whether or not the impairment is confined to the visual domain (and hence difficulties in tasks requiring the manipulation of visual materials) or in the verbal domain (and hence in tasks which are prevalently requiring the maintenance of words).

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Compliance with ethical standards

This research is funded by the Economic and Social Research Council. There are no known conflicts of interest, financial or otherwise, and all data gathered from human participants was done so following obtaining informed consent.

Disclosure statement

No potential conflict of interest was reported by the author(s).

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References

- Allen, K., Giofrè, D., Higgins, S., & Adams, J. (2020). Working memory predictors of written mathematics in 7- to 8-year-old children. *Quarterly Journal of Experimental Psychology*, 73(2), 239–248. <https://doi.org/10.1177/1747021819871243>
- Alloway, T. P., & Alloway, R. G. (2010). Investigating the predictive roles of working memory and IQ in academic attainment. *Journal of Experimental Child Psychology*, 106(1), 20–29. <https://doi.org/10.1016/j.jecp.2009.11.003>
- Andersson, U. (2008). Working memory as a predictor of written arithmetical skills in children: The importance of central executive functions. *The British Journal of Educational Psychology*, 78(Pt 2), 181–203. <https://doi.org/10.1348/000709907X209854>
- Andersson, U., & Lyxell, B. (2007). Working memory deficit in children with mathematical difficulties: A general or specific deficit? *Journal of Experimental Child Psychology*, 96(3), 197–228. <https://doi.org/10.1016/j.jecp.2006.10.001>

- Arsalidou, M., & Taylor, M. J. (2011). Is $2 + 2 = 4$? Meta-analyses of brain areas needed for numbers and calculations. *NeuroImage*, 54(3), 2382–2393. <https://doi.org/10.1016/j.neuroimage.2010.10.009>
- Ashkenazi, S., Rosenberg-Lee, M., Metcalfe, A. W. S., Swigart, A. G., & Menon, V. (2013). Visuo-spatial working memory is an important source of domain-general vulnerability in the development of arithmetic cognition. *Neuropsychologia*, 51(11), 2305–2317. <https://doi.org/10.1016/j.neuropsychologia.2013.06.031>
- Baddeley, A. D. (2000). The episodic buffer: A new component of working memory? *Trends in Cognitive Sciences*, 4(11), 417–423. [https://doi.org/10.1016/S1364-6613\(00\)01538-2](https://doi.org/10.1016/S1364-6613(00)01538-2)
- Baddeley, A. D., & Hitch, G. J. (1974). Working memory. In G. A. Bower (Ed.), *The psychology of learning and motivation: advances in research and theory* (Vol. 8, pp. 47–90). Academic Press.
- Bizzaro, M., Giofrè, D., Girelli, L., & Cornoldi, C. (2018). Arithmetic, working memory, and visuo-spatial imagery abilities in children with poor geometric learning. *Learning and Individual Differences*, 62, 79–88. <https://doi.org/10.1016/j.lindif.2018.01.013>
- Boyer, C. B., & Merzbach, U. C. (2011). *A history of mathematics*. Wiley and Sons.
- Caviola, S., Gerotto, G., & Mammarella, I. C. (2016). Computer-based training for improving mental calculation in third- and fifth-graders. *Acta Psychologica*, 171, 118–127. <https://doi.org/10.1016/j.actpsy.2016.10.005>
- Caviola, S., Mammarella, I. C., Cornoldi, C., & Lucangeli, D. (2012). The involvement of working memory in children's exact and approximate mental addition. *Journal of Experimental Child Psychology*, 112(2), 141–160. <https://doi.org/10.1016/j.jecp.2012.02.005>
- Caviola, S., Mammarella, I. C., Lucangeli, D., & Cornoldi, C. (2014). Working memory and domain-specific precursors predicting success in learning written subtraction problems. *Learning and Individual Differences*, 36, 92–100. <https://doi.org/10.1016/j.lindif.2014.10.010>
- Caviola, S., Mammarella, I. C., Pastore, M., & LeFevre, J.-A. (2018). Children's strategy choices on complex subtraction problems: Individual differences and developmental changes. *Frontiers in Psychology*, 9, 1209. <https://doi.org/10.3389/fpsyg.2018.01209>
- Chuah, Y. M., & Maybery, M. T. (1999). Verbal and spatial short-term memory: Common sources of developmental change? *Journal of Experimental Child Psychology*, 73(1), 7–44. <https://doi.org/10.1006/jecp.1999.2493>
- Clearman, J., Klinger, V., & Szűcs, D. (2017). Visuospatial and verbal memory in mental arithmetic. *Quarterly Journal of Experimental Psychology*, 70(9), 1837–1855. <https://doi.org/10.1080/17470218.2016.1209534>
- Cohen, J., Cohen, P., West, S. G., & Aiken, L. S. (2003). *Applied multiple regression/correlation analysis for the behavioral sciences*. Lawrence Erlbaum Associates.
- Cragg, L., & Gilmore, C. (2014). Skills underlying mathematics: The role of executive function in the development of mathematics proficiency. *Trends in Neuroscience and Education*, 3(2), 63–68. <https://doi.org/10.1016/j.tine.2013.12.001>
- D'Amico, A., & Guarnera, M. (2005). Exploring working memory in children with low arithmetical achievement. *Learning and Individual Differences*, 15(3), 189–202. <https://doi.org/10.1016/j.lindif.2005.01.002>
- De Cruz, H. (2006). Why are some numerical concepts more successful than others? An evolutionary perspective on the history of number concepts. *Evolution and Human Behavior*, 27(4), 306–323. <https://doi.org/10.1016/j.evolhumbehav.2006.02.001>
- Friso-van den Bos, I., van der Ven, S. H. G., Kroesbergen, E. H., & van Luit, J. E. H. (2013). Working memory and mathematics in primary school children: A meta-analysis. *Educational Research Review*, 10, 29–44. <https://doi.org/10.1016/j.edurev.2013.05.003>
- Geary, D. C., Hoard, M. K., Byrd-Craven, J., & DeSoto, C. M. (2004). Strategy choices in simple and complex addition: Contributions of working memory and counting knowledge for children with mathematical disability. *Journal of Experimental Child Psychology*, 88(2), 121–151. <https://doi.org/10.1016/j.jecp.2004.03.002>
- Giofrè, D., Donolato, E., & Mammarella, I. C. (2018). The differential role of verbal and visuospatial working memory in mathematics and reading. *Trends in Neuroscience and Education*, 12(July), 1–6. <https://doi.org/10.1016/j.tine.2018.07.001>

- Giofrè, D., & Mammarella, I. C. (2014). The relationship between working memory and intelligence in children: Is the scoring procedure important? *Intelligence*, 46, 300–310. <https://doi.org/10.1016/j.intell.2014.08.001>
- Giofrè, D., Mammarella, I. C., & Cornoldi, C. (2014). The relationship among geometry, working memory, and intelligence in children. *Journal of Experimental Child Psychology*, 123, 112–128. <https://doi.org/10.1016/J.JECP.2014.01.002>
- Giofrè, D., Mammarella, I. C., Ronconi, L., & Cornoldi, C. (2013). Visuospatial working memory in intuitive geometry, and in academic achievement in geometry. *Learning and Individual Differences*, 23, 114–122. <https://doi.org/10.1016/j.lindif.2012.09.012>
- Grant, H. (1999). Mathematics and the liberal arts. *The College Mathematics Journal*, 30(2), 96–105. <https://doi.org/10.1080/07468342.1999.11974039>
- Heathcote, D. (1994). The role of visuo-spatial working memory in the mental addition of multi-digit addends. *Cahiers de Psychologie Cognitive/Current Psychology of Cognition*, 13(2), 207–245.
- Hitch, G. J., & McAuley, E. (1991). Working memory in children with specific arithmetical learning difficulties. *British Journal of Psychology*, 82(3), 375–386. <https://doi.org/10.1111/j.2044-8295.1991.tb02406.x>
- Holmes, J., & Adams, J. W. (2006). Working memory and children's mathematical skills: Implications for mathematical development and mathematics curricula. *Educational Psychology*, 26(3), 339–366. <https://doi.org/10.1080/01443410500341056>
- Holmes, J., Adams, J. W., & Hamilton, C. J. (2008). The relationship between visuospatial sketch-pad capacity and children's mathematical skills. *European Journal of Cognitive Psychology*, 20(2), 272–289. <https://doi.org/10.1080/09541440701612702>
- Hu, L., & Bentler, P. M. (1999). Cutoff criteria for fit indexes in covariance structure analysis: Conventional criteria versus new alternatives. *Structural Equation Modeling: A Multidisciplinary Journal*, 6(1), 1–55. <https://doi.org/10.1080/10705519909540118>
- Kalaman, D. A., & Lefevre, J.-A. (2007). Working memory demands of exact and approximate addition. *European Journal of Cognitive Psychology*, 19(2), 187–212. <https://doi.org/10.1080/09541440600713445>
- Kanayet, F. J., Mattarella-Micke, A., Kohler, P. J., Norcia, A. M., McCandliss, B. D., & McClelland, J. L. (2018). Distinct representations of magnitude and spatial position within parietal cortex during Number-Space Mapping. *Journal of Cognitive Neuroscience*, 30(2), 200–218. https://doi.org/10.1162/jocn_a_01199
- Kane, M. J., Hambrick, D. Z., Tuholski, S. W., Wilhelm, O., Payne, T. W., & Engle, R. W. (2004). The generality of working memory capacity: A latent-variable approach to verbal and visuospatial memory span and reasoning. *Journal of Experimental Psychology General*, 133(2), 189–217. <https://doi.org/10.1037/0096-3445.133.2.189>
- Katz, I. R., Bennett, R. E., & Berger, A. E. (2000). Effects of response format on difficulty of SAT-mathematics items: It's not the strategy. *Journal of Educational Measurement*, 37(1), 39–57. <https://doi.org/10.1111/j.1745-3984.2000.tb01075.x>
- Kline, R. B. (2011). *Principles and practice of structural equation modeling* (3rd ed.). Guilford Press.
- Krousel-Wood, M., Muntner, P., Jannu, A., Hyre, A., & Breault, J. (2006). Does waiver of written informed consent from the institutional review board affect response rate in a low-risk research study? *Journal of Investigative Medicine*, 54(4), 174 LP–179.
- Kyttaelae, M., & Lehto, J. E. (2008). Some factors underlying mathematical performance: The role of visuospatial working memory and non-verbal intelligence. *European Journal of Psychology of Education*, 23(1), 77–94.
- Li, Y., & Geary, D. C. (2013). Developmental gains in visuospatial memory predict gains in mathematics achievement. *PLOS One*, 8(7), e70160. <https://doi.org/10.1371/journal.pone.0070160>
- Li, Y., & Geary, D. C. (2017). Children's visuospatial memory predicts mathematics achievement through early adolescence. *PLOS One*, 12(2), e0172046. <https://doi.org/10.1371/journal.pone.0172046>
- Mammarella, I. C., Cornoldi, C., Lucangeli, D., Caviola, S., Cornoldi, C., & Lucangeli, D. (2013). Mental additions and verbal-domain interference in children with developmental

- dyscalculia. *Research in Developmental Disabilities*, 34(9), 2845–2855. <https://doi.org/10.1016/j.ridd.2013.05.044>
- Mammarella, I. C., Giofrè, D., Ferrara, R., & Cornoldi, C. (2013). Intuitive geometry and visuospatial working memory in children showing symptoms of nonverbal learning disabilities. *Child Neuropsychology*, 19(3), 235–249. <https://doi.org/10.1080/09297049.2011.640931>
- McLean, J. F., & Hitch, G. J. (1999). Working memory impairments in children with specific arithmetic learning difficulties. *Journal of Experimental Child Psychology*, 74(3), 240–260. <https://doi.org/10.1006/jecp.1999.2516>
- Morra, S., Bisagno, E., Caviola, S., Delfante, C., & Mammarella, I. C. (2019). Working memory capacity and the development of quantitative central conceptual structures. *Cognition and Instruction*, 37(4), 483–511. <https://doi.org/10.1080/07370008.2019.1636797>
- O'Neil Jr, H. F., & Brown, R. S. (1998). Differential effects of question formats in math assessment on metacognition and affect. *Applied Measurement in Education*, 11(4), 331–351. https://doi.org/10.1207/s15324818ame1104_3
- Passolunghi, M. C., & Cornoldi, C. (2008). Working memory failures in children with arithmetical difficulties. *Child Neuropsychology*, 14(5), 387–400. <https://doi.org/10.1080/09297040701566662>
- Passolunghi, M. C., & Mammarella, I. C. (2010). Spatial and visual working memory ability in children with difficulties in arithmetic word problem solving. *European Journal of Cognitive Psychology*, 22(6), 944–963. <https://doi.org/10.1080/09541440903091127>
- Passolunghi, M. C., & Siegel, L. S. (2001). Short-term memory, working memory, and inhibitory control in children with difficulties in arithmetic problem solving. *Journal of Experimental Child Psychology*, 80(1), 44–57. <https://doi.org/10.1006/jecp.2000.2626>
- Pelegrina, S., Capodieci, A., Carretti, B., & Cornoldi, C. (2015). Magnitude representation and working memory updating in children with arithmetic and reading comprehension disabilities. *Journal of Learning Disabilities*, 48(6), 658–668. <https://doi.org/10.1177/0022219414527480>
- Peng, P., Namkung, J., Barnes, M., & Sun, C. (2016). A Meta-Analysis of mathematics and working memory: Moderating effects of working memory domain, type of mathematics skill, and sample characteristics. *Journal of Educational Psychology*, 108(4), 455–473. <https://doi.org/10.1037/edu0000079>
- R Core Team. (2018). *R: A language and environment for statistical computing*. R Foundation for Statistical Computing.
- Raghubar, K. P., Barnes, M. A., & Hecht, S. A. (2010). Working memory and mathematics: A review of developmental, individual difference, and cognitive approaches. *Learning and Individual Differences*, 20(2), 110–122. <https://doi.org/10.1016/j.lindif.2009.10.005>
- Rasmussen, C., & Bisanz, J. (2005). Representation and working memory in early arithmetic. *Journal of Experimental Child Psychology*, 91(2), 137–157. <https://doi.org/10.1016/j.jecp.2005.01.004>
- Revelle, W. (1970, January 01). Psych: Procedures for personality and psychological research. Retrieved September 01, 2020, from <https://www.scholars.northwestern.edu/en/publications/psych-procedures-for-personality-and-psychological-research>
- Rosseel, Y. (2012). lavaan: An R package for structural equation modeling. *Journal of Statistical Software*, 48 (2), 1–36. <https://doi.org/10.18637/jss.v048.i02>
- Shah, P., & Miyake, A. (1996). The separability of working memory resources for spatial thinking and language processing: An individual differences approach. *Journal of Experimental Psychology: General*, 125(1), 4–27. <https://doi.org/10.1037/0096-3445.125.1.4>
- Soltanlou, M., Pixner, S., & Nuerk, H.-C. (2015). Contribution of working memory in multiplication fact network in children may shift from verbal to visuo-spatial: A longitudinal investigation. *Frontiers in Psychology*, 6, 1062. <https://doi.org/10.3389/fpsyg.2015.01062>
- Steel, S., & Funnell, E. (2001). Learning multiplication facts: A study of children taught by discovery methods in England. *Journal of Experimental Child Psychology*, 79(1), 37–55. <https://doi.org/10.1006/jecp.2000.2579>
- Swanson, H. L., & Beebe-Frankenberger, M. (2004). The relationship between working memory and mathematical problem solving in children at risk and not at risk for serious math

- difficulties. *Journal of Educational Psychology*, 96(3), 471–491. <https://doi.org/10.1037/0022-0663.96.3.471>
- Swanson, H. L., & Sachse-Lee, C. (2001). Mathematical problem solving and working memory in children with learning disabilities: Both executive and phonological processes are important. *Journal of Experimental Child Psychology*, 79(3), 294–321. <https://doi.org/10.1006/JECP.2000.2587>
- Szűcs, D., Devine, A., Soltesz, F., Nobes, A., & Gabriel, F. (2013). Developmental dyscalculia is related to visuo-spatial memory and inhibition impairment. *Cortex; a Journal Devoted to the Study of the Nervous System and Behavior*, 49(10), 2674–2688. <https://doi.org/10.1016/j.cortex.2013.06.007>
- Wilson, K. M., & Swanson, H. L. (2001). Are mathematics disabilities due to a domain-general or a domain-specific working memory deficit? *Journal of Learning Disabilities*, 34(3), 237–248. <https://doi.org/10.1177/002221940103400304>
- Wong, B., & Szűcs, D. (2013). Single-digit Arabic numbers do not automatically activate magnitude representations in adults or in children: Evidence from the symbolic same-different task. *Acta Psychologica*, 144(3), 488–498. <https://doi.org/10.1016/j.actpsy.2013.08.006>
- Zheng, X., Swanson, H. L., & Marcoulides, G. A. (2011). Working memory components as predictors of children's mathematical word problem solving. *Journal of Experimental Child Psychology*, 110(4), 481–498. <https://doi.org/10.1016/J.JECP.2011.06.00>